Decoherence of a Josephson qubit due to coupling to two level systems

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Abstract

Noise and decoherence are major obstacles to the implementation of Josephson junction qubits in quantum computing. Recent experiments suggest that two level systems (TLS) in the oxide tunnel barrier are a source of decoherence. We explore two decoherence mechanisms in which these two level systems lead to the decay of Rabi oscillations that result when Josephson junction qubits are subjected to strong microwave driving. (A) We consider a Josephson qubit coupled resonantly to a two level system, i.e., the qubit and TLS have equal energy splittings. As a result of this resonant interaction, the occupation probability of the excited state of the qubit exhibits beating. Decoherence of the qubit results when the two level system decays from its excited state by emitting a phonon. (B) Fluctuations of the two level systems in the oxide barrier produce fluctuations and $1/f$ noise in the Josephson junction critical current I_o. This in turn leads to fluctuations in the qubit energy splitting that degrades the qubit coherence. We compare our results with experiments on Josephson junction phase qubits.

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I. INTRODUCTION

The Josephson junction qubit is a leading candidate as a basic component of a quantum computer. A significant advantage of this approach is scalability, as these qubits may be readily fabricated in large numbers using integrated-circuit technology. Recent experiments on Josephson qubits have successfully shown that they possess quantum coherent properties [1, 2, 3, 4, 5, 6, 7, 8]. However, a major obstacle to the realization of quantum computers with Josephson junction qubits is decoherence and the dominant noise source has not yet been identified. Because the measured decoherence times are substantially shorter than what is needed for a quantum computer, there has been ongoing research to understand decoherence mechanisms in Josephson qubits. Let us briefly review the theoretical work that has been done. Martinis et al. [4] investigated decoherence in Josephson phase qubits due to current noise primarily from external sources. Paladino et al. [9] analyzed decoherence in charge qubits due to background charge fluctuations. Van Harlingen et al. [10, 11] studied how low frequency $1/f$ critical current fluctuations lead to decoherence in various types of Josephson junctions. (f denotes frequency.) Smirnov [12] used perturbation theory to study the decay of Rabi oscillations due to thermal fluctuations of a heat bath weakly coupled to the qubit. Even though fluctuating two level systems in oxide tunnel barriers have long been known to be a major intrinsic noise source in Josephson junctions [13, 14], their role in qubit decoherence has not been investigated theoretically. In this paper we will focus on the intrinsic microscopic mechanisms whereby two level systems produce decoherence in a Josephson qubit.

A recent experiment indicates that two level systems can couple to a Josephson phase qubit [15]. In these experiments the lowest excitation frequency of a qubit is measured as a function of the bias current that determines the depth of the potential energy double well of the qubit. For most values of the current bias, a single excitation frequency ω_{10} is observed and it decreases with increasing bias current. Occasionally the experiments find spurious resonances characterized by two closely spaced excitation frequencies at a given bias current. The size of the gap between the two excitation frequencies is on the order of 25 MHz. Simmonds *et al.* [15] have argued that this splitting is evidence that the qubit is coupled to a two level system (TLS) with an energy splitting very close or equal to that of the qubit. We will refer to this as a resonant interaction. The microscopic nature of

these two level systems is unclear. It may be due to the motion of oxygen atoms in the oxide tunnel barrier of the Josephson junction [16, 17]. An oxygen atom (or vacancy) could sit in a double well potential and tunnel between two positions. Or it may be due to a quasiparticle hopping between two positions in the oxide barrier. Or an electron trap in the oxide barrier could fluctuate between being occupied and empty [13, 14, 16, 18]. Or a trapped flux quantum could be tunneling back and forth between two positions in the oxide barrier. Regardless of the microscopic nature of the TLS, we can use the fact that it is a two level system to understand how such a defect can couple to the qubit. Note that the qubit energy splitting is a function of the critical current I_o. Fluctuations of the TLS lead to fluctuations of the tunneling matrix element T through the oxide barrier. This in turn leads to fluctuations in the critical current I_o since $I_o \sim |T|^2$. If the two states of the TLS correspond to two different values, $I_{o,1}$ and $I_{o,2}$, of the critical current, the coupling between the qubit and the TLS is proportional to the difference $|I_{o,1} - I_{o,2}|$.

Experiments often probe qubits using Rabi oscillations. Let us take a moment to review Rabi oscillations [19]. If the qubit is initially in its ground state, resonant microwaves with a frequency that matches the qubit energy splitting (ω_{10}) will initially increase the probability amplitude of finding the qubit in its excited state ($|1\rangle$). However, as time goes on, at some point the qubit is completely in its excited state, and the electromagnetic wave goes on to de-excite the qubit through stimulated emission. Thus the system will be coherently oscillating between the two energy eigenstates with a Rabi frequency f_R. The frequency f_R of the Rabi oscillations increases linearly with the amplitude of the driving electric field. Rabi oscillations have been seen in the occupation probability P_1 of the excited state of a Josephson qubit [19]. This demonstration of quantum coherence is a preliminary requirement for quantum computing but most of the reported Rabi oscillations in Josephson qubits have a rather small amplitude (less than 50%) and a short coherence time (less than one microsecond). Experiments have found that the presence of a resonant interaction between the qubit and a two level system substantially reduces or even eliminates Rabi oscillations [15].

In this paper we theoretically model a qubit coupled to a two level system and study the effect of this coupling on Rabi oscillations. We study the quantum dynamics of a Josephson qubit by numerically integrating the time-dependent Schrödinger equation. Our method is not limited to weak coupling to the noise sources. While our analysis applies to any Josephson qubit, for illustration we consider the Josephson phase qubit that was

studied in recent experiments [15]. We explore two decoherence mechanisms where two level systems lead to the decay of Rabi oscillations that result when Josephson junction qubits are subjected to strong microwave driving. We first consider the resonant case. In the resonant regime, the energy splitting of the two level system and the qubit are matched. As a result the occupation probability of the qubit's excited state exhibits beating. This has been termed a qubit duet [7, 8]. Our calculations show that decoherence of the qubit results when the two level system decays from its excited state by emitting a phonon. In section II, we numerically calculate the qubit occupation probability as a function of time and compare it with experiment.

The other case involves low frequency fluctuations of the qubit energy splitting. Even though the qubit energy splitting $\omega_{10}/2\pi$ is on the order 10 GHz, low frequency fluctuations of the critical current lead to low frequency fluctuations of the qubit energy splitting. We hypothesize that this noise comes from slow fluctuations of two level systems in the oxide tunnel barrier. If we assume that there are a number of two level systems in the barrier, and if these two level systems have a broad distribution of decay rates, then they will produce $1/f$ noise [20, 21] that leads to decoherence. In section III we show that these fluctuations in the qubit energy splitting lead to decay of the Rabi oscillations. We consider three cases. In the first case the qubit is coupled to a single slowly fluctuating two level system. In the second case the qubit is coupled resonantly to a two level system and, at the same time, is subjected to slow fluctuations in the critical current of a two level system. In the third case we consider a qubit with energy splitting fluctuations that have a $1/f$ noise spectrum.

II. QUBIT-TLS RESONANCE

In our model we assume that there are two level systems in the oxide barrier of the Josephson junction. The standard model of noninteracting two level systems [22, 23] was introduced by Anderson, Halperin, and Varma [24], and independently by W. A. Phillips[25] in 1972. The standard Hamiltonian for a two level system is

$$H = \frac{1}{2}\begin{pmatrix} \Delta & \Delta_o \\ -\Delta_o & -\Delta \end{pmatrix} . \tag{1}$$

Here we are using the left well – right well basis where $|L\rangle$ ($|R\rangle$) is the left (right) well state. Δ is the asymmetry energy, i.e., Δ is the energy difference between the right well and the

left well. We can diagonalize the Hamiltonian to get the energy eigenvalues that are given by $\pm\varepsilon_{TLS}/2$ where

$$\varepsilon_{TLS} = \sqrt{\Delta^2 + \Delta_o^2}\,. \tag{2}$$

Simmonds *et al.* have found experimental evidence for resonant interactions between a phase qubit and a two level system [15]. This resonant interaction occurs when the energy splitting ϵ_{TLS} of the TLS matches that of the qubit, i.e., $\epsilon_{TLS} = \hbar\omega_{10}$. Simmonds *et al.* constructed a phenomenological model to account for their experimental findings [15]. In this section we investigate Rabi oscillations in the presence of a qubit-TLS resonance. We incorporate decoherence into the model of Simmonds *et al.* to show that Rabi oscillations can decohere due to decay of the excited state of the two level system via phonon emission. We briefly describe the model of Simmonds *et al.* in the following.

The Hamiltonian of a Josephson phase qubit (which is essentially a current-biased Josephson junction) is [26, 27, 28]

$$H_{qb} = \frac{\hat{Q}^2}{2C} - \frac{\Phi_0 I_o}{2\pi}\cos\hat{\delta} - \frac{\Phi_0 I_{bias}}{2\pi}\hat{\delta} \tag{3}$$

where $\Phi_0 = h/2e$ is a superconducting flux quantum. C is the capacitance of the junction, and I_{bias} is the bias current. The operators $\hat{Q}$ and $\hat{\delta}$ correspond to the charge and phase difference across the Josephson junction respectively.

Next we assume that there is a two level system in the barrier of the Josephson junction. The two states of the TLS correspond to two different values of the Josephson junction critical current I_o which is proportional to the square of the tunneling matrix element. When the TLS is in state $|R\rangle$ (state $|L\rangle$), the junction critical current is I_{0R} (I_{0L}). The qubit couples to the TLS because the qubit's energy splitting $\hbar\omega_{10}$ is a function of I_o. The expression for ω_{10} can be derived from the Hamiltonian in Eq. (3) using the expression for the resonant frequency of an LC circuit $\omega_{10} \approx 1/\sqrt{L_J C}$ where $L_J = \Phi_o/2\pi I_o \cos\delta$ is the Josephson inductance [29]:

$$\omega_{10} \approx \sqrt{\frac{2\pi I_o}{\Phi_0 C}}\left[2\left(1 - \frac{I_{bias}}{I_o}\right)\right]^{1/4}. \tag{4}$$

Here we used the fact that $I = I_o \sin\delta$ implies that $\cos\delta = \sqrt{1-(I/I_o)^2}$. Typically I_{bias} is slightly less than I_o. The qubit couples to the TLS because the Josephson junction critical current I_o is modified by the TLS. Therefore, the interaction Hamiltonian is [15]:

$$H_{qb-TLS} = -\frac{\Phi_0 I_{0R}}{2\pi}\cos\delta \otimes |R\rangle\langle R| - \frac{\Phi_0 I_{0L}}{2\pi}\cos\delta \otimes |L\rangle\langle L|\,. \tag{5}$$

We can transform to the eigenbasis of the TLS. Provided that the TLS is symmetric, its ground state is $|g\rangle = (|R\rangle + |L\rangle)/\sqrt{2}$ and its excited state is $|e\rangle = (|R\rangle - |L\rangle)/\sqrt{2}$. The ground state of the qubit is $|0\rangle$ and the excited state of the qubit is $|1\rangle$. We can rewrite the operator $\cos\hat{\delta}$ in terms of its matrix elements in the qubit basis by noting that the phase qubit is typically biased close to $\delta = \pi/2$ where the Josephson current is maximized. So $\cos(\pi/2 - \delta') = \sin(\delta') \approx \delta'$ where $\delta' \ll 1$. δ' can be represented as a sum of a creation and an annihilation operator in much the same way as the position coordinate of a harmonic oscillator. Using these facts, we find that Eq. (5) becomes [15]

$$H_{qb-TLS} = \frac{\delta I_o}{2}\sqrt{\frac{\hbar}{2\omega_{10}C}}\left(|0,g\rangle\langle 1,e| + |1,e\rangle\langle 0,g| + |1,g\rangle\langle 0,e| + |0,e\rangle\langle 1,g|\right), \tag{6}$$

where ω_{10} is the energy difference in the qubit levels, and $\delta I_o \equiv I_{0R} - I_{0L}$ is the fluctuation amplitude in I_o produced by the TLS. Since the values of $\langle H_{qb-TLS}\rangle$, C and ω_{10} can be determined experimentally, one can estimate $\delta I_o/I_o$ to be approximately 6×10^{-5} [15] when the Josephson junction is in the zero-voltage state. However this value of $\delta I_o/I_o$ differs from previous measurements made on Josephson junctions in the finite voltage state that found low frequency (1 kHz or less) fluctuations with $\delta I_o/I_o \sim 2 \times 10^{-6}$ [10, 13].

Next we study the quantum dynamics of the coupled qubit-TLS system when it is subjected to microwave driving at a frequency ω_{10} equal to the qubit energy splitting. We will assume that both the qubit and the TLS can couple to microwaves. The resulting Hamiltonian matrix of the qubit-TLS model is,

$$H_{qb-TLS} = \begin{pmatrix} 0 & g_{TLS}\sin(\omega_{10}t) & g_{qb}\sin(\omega_{10}t) & \eta \\ g_{TLS}\sin(\omega_{10}t) & \varepsilon_{TLS} & \eta & g_{qb}\sin(\omega_{10}t) \\ g_{qb}\sin(\omega_{10}t) & \eta & \hbar\omega_{10} & g_{TLS}\sin(\omega_{10}t) \\ \eta & g_{qb}\sin(\omega_{10}t) & g_{TLS}\sin(\omega_{10}t) & \hbar\omega_{10} + \varepsilon_{TLS} \end{pmatrix} \tag{7}$$

where the basis states are $|0,g\rangle$, $|0,e\rangle$, $|1,g\rangle$, and $|1,e\rangle$ respectively. $\eta = (\delta I_o/2)\sqrt{\hbar/(2\omega_{10}C)}$ is the coupling between the qubit and the TLS; g_{qb} is the coupling between the qubit and the microwaves; and g_{TLS} is the coupling between the TLS and the microwaves. In the strong driving regime, the qubit-microwave coupling is larger than the qubit-TLS coupling. We consider the case of strong driving throughout the paper because it is the experimental condition in Ref. [15]. Without microwave driving, the Hamiltonian matrix can be decoupled into two 2×2 matrices. We calculate the time evolution of the qubit-TLS wave function by

integrating the time-dependent Schrödinger equation using the Runga-Kutta method[30]. The initial wave function $|\psi\rangle$ is the ground state of Eq. (7). When $\hbar\omega_{10}$ and ε_{TLS} are much greater than η, the ground state wave function is approximately given by $|0, g\rangle$.

We first consider the case of strong driving with $g_{TLS} = 0$ and with the TLS in resonance with the qubit, i.e. $\varepsilon_{TLS} = \hbar\omega_{10}$. If there is no coupling between the qubit and the TLS, then the four states of the system are the ground state $|0, g\rangle$, the highest energy state $|1, e\rangle$, and two degenerate states in the middle $|1, g\rangle$ and $|0, e\rangle$. If the qubit and the TLS are coupled with coupling strength η, the degeneracy is split by an energy 2η. Figure 1 shows the coherent oscillations of the resonant qubit-TLS system. We define a projection operator $\hat{P}_1 \equiv |1, g\rangle\langle 1, g| + |1, e\rangle\langle 1, e|$ so that $\langle\hat{P}_1\rangle$ corresponds to the occupation probability of the qubit to be in state $|1\rangle$ as in the phase-qubit experiment. Instead of being sinusoidal like typical Rabi oscillations (the dotted curve), the occupation probability P_1 exhibits beating (Fig. 1a) because the two entangled states that are linear combinations of $|1, g\rangle$ and $|0, e\rangle$ have a small energy splitting 2η, and this small splitting is the beat frequency. Without any source of decoherence, the resonant beating will not decay. Thus far the beating phenomenon has not yet been experimentally verified. The lack of experimental observation of beating implies that the TLS or qubit decoheres in less time than the period $\approx 1/\eta$. Note in Fig. 1a that the second beat is out of phase when compared with the usual Rabi oscillations. The occupation probabilities in the individual states are plotted in Fig. 1b-1e. (To the best of our knowledge, these quantities are not measurable.) We find that there is a very low occupation probability in the states $|0, e\rangle$ and $|1, e\rangle$ during the first Rabi cycle because the TLS is not directly coupled to the microwaves, and thus the TLS tends to be in its ground state $|g\rangle$. In the limit $\eta \to 0$, the system oscillates coherently between $|0, g\rangle$ and $|1, g\rangle$. Occupying the two states $|0, e\rangle$ and $|1, e\rangle$ occurs only via the qubit-TLS resonance coupling η. From Fermi's golden rule, the average transition rate to $|0, e\rangle$ from $|1, g\rangle$ is $2\pi\eta^2/\hbar$ which is much slower than the initial transition rate from $|0, g\rangle$ to $|1, g\rangle$.

Now we consider the decay of excited TLS via phonons as a source of decoherence for the qubit-TLS system. Two level systems couple to the strain field and are able to decay from their excited state by emitting a phonon with an energy equal to the TLS energy splitting. The rate for an excited TLS to emit a phonon and return to its ground state is given by [22]

$$\tau_{ph}^{-1} = \frac{\tilde{\gamma}^2}{\rho}\left(\frac{1}{c_l^5} + \frac{2}{c_t^5}\right)\frac{\varepsilon_{TLS}^3}{2\pi\hbar^4}\left(\frac{\Delta_0}{\varepsilon_{TLS}}\right)^2 \coth\left(\frac{\beta\varepsilon_{TLS}}{2}\right) \tag{8}$$

where $\tilde{\gamma}$ is the deformation potential, ρ is the mass density, c_l (c_t) is the longitudinal (transverse) speed of sound, and β is the inverse temperature. From Eq. (8), the relaxation time τ_{ph} is estimated to be in the range from 10 to 100 ns. (We find $\tau_{ph} \sim 80$ ns when using the following values that are appropriate for a symmetric TLS in SiO_2 with a tunnel splitting that matches the energy splitting of a Josephson junction qubit: $\tilde{\gamma} = 1.0$ eV, $\rho = 2.2$ g/cm^3, $c_l = 5.8 \times 10^5$ cm/s, $c_t = 3.8 \times 10^5$ cm/s, $\Delta = 0$, $\Delta_o = 0.5K$, and $T = 25$ mK.)

The decay of the excited two level system leads to decoherence of the Rabi oscillations. We can incorporate this relaxation rate of the excited TLS into our calculations of the Rabi oscillations of the qubit-TLS system by using the Monte Carlo wave function method [31]. In the original application [31] a two-level atom driven by a laser field can decay by emitting a photon. We can easily generalize the algorithm for our somewhat more complicated case in which the qubit-TLS system decays either from $|0,e\rangle$ to $|0,g\rangle$ or from $|1,e\rangle$ to $|1,g\rangle$. The algorithm goes as follows. (a) Numerically propagate the wave function from time t_i to $t_i + \Delta t$ as if there is no energy decay in the TLS. Note that the probability for decaying from $|0,e\rangle$ to $|0,g\rangle$ during the time interval Δt is $P_{(0,e)} \times \left[1 - \exp\left(\frac{-\Delta t}{\tau_{ph}}\right)\right]$, where $P_{(0,e)}$ is the probability that state $|0,e\rangle$ is occupied. (b) Generate a uniformly distributed random number $r_i \in [0,1]$. If $r_i < P_{(0,e)} \times \left[1 - \exp\left(\frac{-\Delta t}{\tau_{ph}}\right)\right]$, then the system decays from $|0,e\rangle$ to $|0,g\rangle$, and we represent this by resetting the wave function $\psi(t_i + \Delta t)$ to $|0,g\rangle$. Then we repeat steps (a) and (b). Similarly, if $1 - r_i < P_{(1,e)} \times \left[1 - \exp\left(\frac{-\Delta t}{\tau_{ph}}\right)\right]$, then the system decays from $|1,e\rangle$ to $|1,g\rangle$, and we represent this by resetting the wave function $\psi(t_i + \Delta t)$ to $|1,g\rangle$. Then we repeat steps (a)-(b). Notice that $\left[1 - \exp\left(\frac{-\Delta t}{\tau_{ph}}\right)\right]$ is very small since the time step Δt is small. So there is no chance that both decays could happen in the same time step. If neither of the above criteria are satisfied, then the qubit does not decay. Then we repeat steps (a)-(b) and keep propagating the wave function until the desired finishing time.

We have used our algorithm to study the effect of TLS energy decay via phonon emission on Rabi oscillations. The results are shown in Fig. 2. The dotted lines show the beating that occurs when the qubit and TLS are in resonance with no TLS decay. The solid lines shows the rapid damping of the Rabi oscillations and the dephasing that occurs when the TLS can decay via phonon emission. In Fig. 2a there is no direct coupling between the microwaves and the TLS, whereas in Fig. 2b, the microwaves are directly coupled both to the qubit and to the TLS. We see that in the latter case beating is damped out more quickly. The

Rabi decay time τ_{Rabi} can be defined as the time for the envelope of the Rabi oscillations to decay by 1/e of their original amplitude. Fig. 2 shows that τ_{Rabi} is longer than τ_{ph} simply because the excited state of the TLS is not always occupied and available for decay. Fig. 1 shows that states $|0,e\rangle$ and $|1,e\rangle$ are essentially unoccupied at early times and that the TLS energy decay can take place only when the excited state of the TLS is sufficiently populated. Therefore in Fig. 2a the solid and dotted lines are in phase for the first few Rabi cycles.

Figure 2b shows the effect of coupling between the microwaves and the TLS. It has been shown experimentally that a TLS will couple to microwaves if it has an intrinsic or induced dipole moment [32, 33]. We can estimate the value of the matrix element g_{TLS} to be approximately pE where p is the magnitude of the dipole moment and E is the magnitude of the electric field produced by the incident microwaves. We can estimate p by assuming that the TLS is a charged particle (with charge $|q| = e$) hopping between two sites separated by a couple of angstroms. We can estimate $E \sim V/d$ where $d \sim 20\mathring{A}$ is the thickness of the oxide barrier and V is the voltage produced across the junction by microwaves. We can calculate V using the Josephson equations with the current being the microwave current $I_{\mu w}$ across the junction. Using the experimental values from Ref. [15], we can estimate $I_{\mu w} \approx 1$ pA. Therefore, we have $g_{TLS} \approx 0.01\hbar\omega_{10}$, which is comparable to g_{qb}. Comparing the two panels, we find that the TLS-microwave coupling greatly enhances the resonant decoherence mechanism because the transition rates from the ground state to the states $|0,e\rangle$ and $|1,e\rangle$ increase as g_{TLS} increases. We conclude that the energy decay of the TLS mainly causes the Rabi oscillations to decay, and adding the TLS-microwave coupling further degrades the qubit coherence. However, the phase qubit experiment indicates that the resonant interaction affects both the Rabi amplitude and decay time [34]. The large amplitude of the Rabi oscillations that we see at short times is not seen experimentally. Experimentally, a qubit in resonance with a TLS has very small amplitude Rabi oscillations at all times. This implies that our calculations do not include all the sources of decoherence responsible for the experimental observations, such as resonance with multiple fluctuators and interactions between the fluctuators.

Next we propose an experiment to test if the TLS in the Josephson qubit couples to microwaves or not. Start in the ground state, then send in a resonant microwave π-pulse and stop pumping. (A π-pulse lasts for half of a Rabi cycle.) When $g_{TLS} \ll g_{qb}$, the qubit-TLS system mainly occupies the state $|1,g\rangle$ right after the π pulse. State $|1,g\rangle$ is a

superposition of two eigenstates ($|\psi_1'\rangle = (|0,e\rangle + |1,g\rangle)/\sqrt{2}$ and $|\psi_2'\rangle = (|0,e\rangle - |1,g\rangle)/\sqrt{2}$) of the qubit-TLS system. Thus the wave function oscillates between the states $|0,e\rangle$ and $|1,g\rangle$ after the π-pulse. Provided that the relaxation time of the TLS is longer than $1/\eta$, coherent oscillations in the occupation probability P_1 can be observed with a period of 2η [35]. If there is no energy decay of the excited TLS, there is no mechanism for decoherence and the oscillation amplitude is one. When g_{TLS} is comparable to g_{qb}, both states $|1,g\rangle$ and $|1,e\rangle$ are partially occupied right after the π-pulse. State $|1,e\rangle$ is essentially an stationary state. Thus partially occupying $|1,e\rangle$ reduces the Rabi oscillation amplitude of P_1 and merely gives a constant contribution to P_1. Figure 3 shows the dynamics of the qubit-TLS system during and after a microwave π-pulse. As we expect, Fig. 3a shows that the oscillation amplitude decreases as g_{TLS} increases. In Fig. 3a the excited TLS has no means of decay ($\tau_{ph} = \infty$), so the coherent oscillations do not decay. In Fig. 3b and Fig. 3c, we set $\tau_{ph} = 40$ ns. As a result, the oscillatory P_1 is attenuated. We include the effect of the energy decay of the TLS by using the Monte Carlo wave function method described previously. These model calculations suggest that one can estimate g_{TLS} by measuring the oscillation amplitude. An alternative way to measure g_{TLS} is to compare the Rabi oscillations after π and 3π pulses. If $g_{TLS} = 0$, right after a π or 3π pulse, the system primarily occupies the state $|1,g\rangle$ with $P_{(1,g)} \sim 1$. Therefore the coherent oscillations after the π and 3π pulses ought to be similar. However, if g_{TLS} is nonzero, the longer pulse will pump more weight into state $|0,e\rangle$. So the amplitude of the oscillations of P_1 right after a π-pulse will be greater than immediately after a 3π-pulse. With a 3π-pulse, the oscillations in P_1 are more damped, as shown in Fig. 3c.

III. THE LOW-FREQUENCY TWO-LEVEL FLUCTUATOR

As described in Sec. I, a two level system trapped inside a Josephson junction barrier can produce noise in the critical current I_o by varying the height of the tunneling potential barrier. As a result, the qubit energy levels fluctuate and this leads to decoherence. We now study the decoherence produced by a low-frequency TLS. As we mentioned earlier, the dependence of ω_{10} on I_o is given by Eq. 4. We note that ω_{10} is modulated as I_o varies. As the TLS modulates I_o, the phase of the qubit is randomized and coherent temporal oscillations

are destroyed. Using the fact that I_{bias} is slightly smaller than I_o in Eq. (4), one finds

$$\frac{\delta\omega_{10}}{\langle\omega_{10}\rangle} \approx \frac{\delta I_o}{4\left(\langle I_o\rangle - I_{bias}\right)} \ . \tag{9}$$

In Ref. [15], it was found that $\delta I/\langle I_o\rangle \approx 6\times 10^{-5}$. In addition, the phase qubit is typically operated at a bias current such that $(\langle I_o\rangle - I_{bias})/\langle I_o\rangle = 0.0025$. Substituting these numbers into Eq. (9), one can estimate that the amplitude of the level fluctuations $\delta\omega_{10}/\langle\omega_{10}\rangle$ is approximately 0.006.

The Rabi oscillations are calculated by integrating Schrödinger's equation in the presence of noise from a single low-frequency fluctuator, i.e., $\omega_{10}t_{TLS} \ll 1$. t_{TLS} is the characteristic time of the random switching of the TLS. We simulate the noise using the Monte Carlo method. At each time t_i, a random number $r_i \in [0,1]$ is generated. If $r_i < ((\text{time step})/t_{TLS}) < 1$, then the two level system switches wells. We can imagine that these are thermally activated transitions between the two wells of a symmetric double well potential. We assume here that the dwell times t_{TLS} are the same in the two wells. Each time the TLS switches wells, the critical current, and hence ω_{10} switch between two values. The Rabi oscillations of the qubit are calculated using the Hamiltonian

$$H(t) = \begin{pmatrix} 0 & g_{qb}\sin(\omega_{10}t) \\ g_{qb}\sin(\omega_{10}t) & \hbar[\omega_{10} + \delta\omega_{10}(t)] \end{pmatrix} \tag{10}$$

where the qubit energy levels ($|0\rangle$ and $|1\rangle$) are the basis states and the noise is produced by a single TLS. Our calculations are oriented to the experimental conditions and the results are shown in Fig. 4. In Fig. 4a-c the characteristic fluctuation rate $t_{TLS}^{-1} = 0.6$ GHz. Panel 4a shows that the qubit essentially stays coherent when the level fluctuations are small ($\delta\omega_{10}/\omega_{10} = 0.001$). Panel 4a shows that when the level fluctuations increase to 0.006, the Rabi oscillations decay within 100 ns. The Rabi relaxation time also depends on the Rabi frequency as panel 4c shows. The faster the Rabi oscillations, the longer they last. This is because the low-frequency noise is essentially constant over several rapid Rabi oscillations [12]. Alternatively, one can explain it by the noise power spectrum $S_I(f)$. Since the noise from a single TLS is a random process characterized by a single characteristic time scale t_{TLS}, it has a Lorentzian power spectrum [36, 37, 38, 39]

$$S_I(f) \sim \frac{t_{TLS}}{(2\pi f t_{TLS})^2 + 1} \tag{11}$$

We do not expect Rabi oscillations to be sensitive to noise at frequencies much greater than the frequency of the Rabi oscillations because the higher the frequency f, the smaller the noise power and because the Rabi oscillations will tend to average over the noise. Rabi dynamics are sensitive to the noise at frequencies comparable to the Rabi frequency. In addition, the characteristic fluctuation rate plays an important role in the rate of relaxation of the Rabi oscillations. It has been shown that t_{TLS}^{-1} can be thermally activated [40] for TLS in a metal-insulator-metal tunnel junction. If the thermally activated behavior applies here, the decoherence time τ_{Rabi} should decrease as temperature increases. In Fig. 4d, the characteristic fluctuation rate has been lowered to 0.06 GHz (which is much lower than $\omega_{10}/2\pi \approx 10$ GHz). The noise still causes qubit decoherence but affects the qubit less than in Fig. 4c. Fig. 4 shows that the noise primarily affects the Rabi amplitude rather than the phase.

Experimentally, the two TLS decoherence mechanisms (resonant interaction and low-frequency level fluctuations) can both be active at the same time. We have calculated the Rabi oscillations in the presence of both of these decoherence sources by using the qubit-TLS Hamiltonian in eq. (7) with a fluctuating $\omega_{10}(t)$ that is generated in the same way and with the same amplitude as in Figure 4b. We show the result in Fig. 5. By comparing Fig. 5 with Fig. 2b, we note that adding level fluctuations reduces the Rabi amplitude and renormalizes the Rabi frequency. The result in Fig. 5 is closer to what is seen experimentally [15].

In a charge qubit, the qubit energy splitting is $\hbar I_o/2e$ when biased at the degeneracy point [2]. So for a charge qubit critical current fluctuations are related to fluctuations in the qubit energy splitting by

$$\frac{\delta\omega_{10}}{\langle\omega_{10}\rangle} = \frac{\delta I_o}{\langle I_o\rangle} \ . \tag{12}$$

Comparing this to the analogous expression for a phase qubit (Eq. (9)), it is obvious that for the same amount of noise in the critical current $\delta I_o/\langle I_o\rangle$, the level fluctuations $\delta\omega_{10}/\omega_{10}$ of a charge qubit seem to be less than those of a phase qubit. This is because the phase qubit must be biased with a current slightly less than $\langle I_o\rangle$, which magnifies the critical current noise typically by a factor of 100. However, the area of the Josephson junctions in charge qubits is much smaller than in phase qubits. As a result, $\langle I_o\rangle$ is smaller in the charge qubit. (In Ref. [2], the charge qubit has $I_o \approx 30$ nA. In Ref. [15], the phase qubit has $I_o \approx 11.6$ μA.) Thus for a fixed value of δI_o, the charge qubit has a much larger relative fluctuation $\delta I_o/\langle I_o\rangle$. Therefore we believe that level fluctuations remain a concern for charge qubits.

1/f noise in the critical current of Josephson junctions has been observed experimentally [41, 42, 43, 44]. The microscopic source of this noise is unknown, though it has been suggested that the tunneling of atoms or ions is involved [16]. When an ensemble of two level systems with a distribution of relaxation times is considered, we can obtain the resulting noise power spectrum by averaging the Lorentzian power spectra of individual TLS over the distribution of relaxation times [20]. For example we can replace t_{TLS} in the Lorentzian in Eq. (11) with the TLS relaxation rate τ_{TLS} given by Eq. (8), and average over the TLS parameters Δ and Δ_o [21]. The result is a 1/f noise spectrum in the critical current, and hence in the qubit level fluctuations $\delta\omega_{10}$. So we have considered the effect of 1/f noise in a Josephson junction on qubit decoherence. We do not average over Lorentzians to obtain our noise spectrum. Rather we numerically generate a time series $\delta\omega_{10}$ with a 1/f noise power spectrum. We use this $\delta\omega_{10}$ in Eq. (10) and calculate the Rabi oscillations with the same numerical approach as in Fig. 4. The results are shown in Fig. 6. When the noise power is low, the Rabi amplitude is reduced but it remains in phase with the unperturbed Rabi oscillations (dotted line). However, as Fig. 6b shows, the coherent nature will eventually be destroyed as the noise power increases. The decaying Rabi oscillations clearly demonstrate that 1/f noise is able to adversely affect the coherence of the qubit.

IV. CONCLUSION

We have shown that the coupling between a two-level fluctuator and a Josephson qubit has a large effect on the decay and decoherence of the Rabi oscillations. We have studied the quantum dynamics of a Josephson qubit subjected to microwave driving by numerically integrating the time-dependent Schrödinger equation. We focussed on two decoherence mechanisms of a Josephson qubit caused by coupling to two level systems. (A) In the resonant regime we considered a qubit in resonance with a high-frequency TLS, i.e., a TLS with a large tunneling splitting. Without any source of energy decay, the qubit occupation probability P_1 of the excited state exhibits beating rather than being sinusoidal as in the usual Rabi oscillations. Including the energy decay of the excited TLS via phonon emission results in the decoherence of the coupled qubit-TLS system. This decoherence mechanism primarily causes a characteristic relaxation time of the Rabi oscillations. The Rabi dynamics at short times is not affected. Furthermore, coupling between the TLS and the microwave

driving is found to further degrade qubit coherence. (B) The other regime involved qubit level fluctuations. Low-frequency TLS are treated as noise sources because they randomly modulate the junction critical current I_o. Fluctuations in I_o modulate the qubit energy splitting, thus randomizing the phase of the qubit, leading to decoherence. Based on noise measurements in prior experiments, our model calculations suggest that noise from a single TLS can cause Rabi oscillations to decay within 100 ns. When the qubit is coupled to a single slow fluctuator, we have shown that the Rabi decay time depends on the noise amplitude $\delta\omega_{10}$, the characteristic fluctuation rate, and the Rabi frequency. When the qubit level fluctuations have a $1/f$ noise spectrum, the Rabi oscillation degradation increases with increasing noise power.

V. ACKNOWLEDGEMENT

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[1] For a comprehensive review in fundamental principles and engineering in Josephson qubit, see Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).

[2] Y. Nakamura, Y. Pashkin, and J. S. Tsai, Nature **398**, 768 (1999).

[3] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. Devoret, Science **296**, 886 (2002).

[4] J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901 (2002).

[5] Y. Yu, S. Han, X. Chu, S.-I. Chu, and Z. Wang, Science **296**, 889 (2003).

[6] I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooji, Science **299**, 1869 (2003).

[7] Y. A. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D. V. Averin, and J. S. Tsai, Nature **421**, 823 (2003).

[8] A. J. Berkley, H. Xu, R. C. Ramos, M. A. Gubrud, F. W. Strauch, P. R. Johnson, J. R. Anderson, A. J. Dragt, C. J. Lobb, and F. C. Wellstood, Science **300**, 1548 (2003).

[9] E. Paladino, L. Faoro, G. Falci, and R. Fazio (2002).

[10] D. J. VanHarlingen, B. L. T. Plourde, T. L. Robertson, P. A. Reichardt, and J. Clarke, in *Quantum Computing and Quantum Bits in Mesoscopic Systems*, edited by A. J. Leggett, B. Ruggiero, and P. Silvestrini (Kluwer Academic/Plenum, New York, 2004).

[11] D. J. VanHarlingen, T. L. Robertson, B. L. T. Plourde, P. A. Reichardt, T. A. Crane, and J. Clarke, Phys. Rev. B **70**, 064517 (2004).

[12] A. Y. Smirnov, Phys. Rev. B **67**, 155104 (2003).

[13] R. T. Wakai and D. J. VanHarlingen, Appl. Phys. Lett. **49**, 593 (1986).

[14] R. T. Wakai and D. J. VanHarlingen, Phys. Rev. Lett. **58**, 1687 (1987).

[15] R. W. Simmonds, K. M. Lang, D. A. Hite, D. P. Pappas, and J. M. Martinis, Phys. Rev. Lett. **93** (2004), to be published, cond-mat/0402470.

[16] C. T. Rogers and R. A. Buhrman, Phys. Rev. Lett. **55**, 859 (1985).

[17] R. A. Buhrman, private communication.

[18] C. T. Rogers and R. A. Buhrman, Phys. Rev. Lett. **53**, 1272 (1984).

[19] I. I. Rabi, Phys. Rev. **51**, 652 (1937).

[20] P. Dutta and P. M. Horn, Rev. Mod. Phys. **53**, 497 (1981).

[21] C. C. Yu, J. Low Temp. Phys. **137** (2004), to be published.

[22] S. Hunklinger and A. K. Raychaudhuri, Prog. in Low Temp. Phys. **9**, 265 (1986).

[23] W. A. Phillips, *Amorphous Solids* (Springer–Verlag, New York, 1981).

[24] P. Anderson, B. Halperin, and C. Varma, Philos. Mag. **25**, 1 (1972).

[25] W. A. Phillips, J. Low Temp. Phys. **7**, 351 (1972).

[26] A. J. Leggett, in *Chance and Matter*, edited by J. Souletie, J. Vannimenus, and R. Stora (Elsevier, Amsterdam, 1987), p. 395.

[27] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).

[28] J. Clarke, A. N. Cleland, M. H. Devoret, D. Esteve, and J. M. Martinis, Science **239**, 992 (1988).

[29] M. Steffen, J. M. Martinis, and I. L. Chuang, Phys. Rev. B **68**, 224518 (2003).

[30] We use Runge-Kutta method to integrate Schödinger's equation. The time step is small enough to conserve the norm of the evolving wave function. The error in the norm is set to be less than under 10^{-6} over 10^7 time steps. Details of the Runge-Kutta method can be found in W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes*, (Cambridge University Press, New York, 1986).

[31] J. Dalibard, Y. Castin, and K. Molmer, Phys. Rev. Lett. **68**, 580 (1992).

[32] B. Golding, M. v. Schickfus, S. Hunklinger, and K. Dransfeld, Phys. Rev. Lett. **43**, 1817 (1979).

[33] B. Golding, J. E. Graebner, and W. H. Haemmerle, Phys. Rev. Lett. **44**, 899 (1980).

[34] J. M. Martinis, private communication.

[35] Very recently, the coherent oscillations after a short pulse have been successfully detected using an improved qubit readout technique. K. B. Cooper, M. Steffen, R. McDermott, R. W. Simmonds, S. Oh, D. A. Hite, D. P. Pappas, and J. M. Martinis, cond-mat/0405710.

[36] S. Machlup, J. Appl. Phys. **25**, 241 (1954).

[37] J. Bernamont, Ann. Phys. (Leipzig) **7**, 71 (1937).

[38] F. K. du Pré, Phys. Rev. **78**, 615 (1950).

[39] A. V. der Ziel, Physica **16**, 359 (1950).

[40] The thermally activated behavior $t_{TLF}^{-1} = (1/t_0)\exp(-E_a/k_BT)$ was experimentally verified in Ref. [18] for $T \geq 15K$. However since we are considering a much lower temperature, the above relation might not hold.

[41] D. J. VanHarlingen, R. H. Koch, and J. Clarke, Appl. Phys. Lett. **41**, 197 (1982).

[42] V. Foglietti, W. J. Gallagher, M. B. Ketchen, A. W. Kleinsasser, R. H. Koch, S. I. Raider, and R. L. Sandstrom, Appl. Phys. Lett. **49**, 1393 (1986).

[43] B. Savo, F. C. Wellstood, and J. Clarke, Appl. Phys. Lett. **50**, 1757 (1987).

[44] F. C. Wellstood, Ph.D. thesis, University of California, Berkeley (1988).

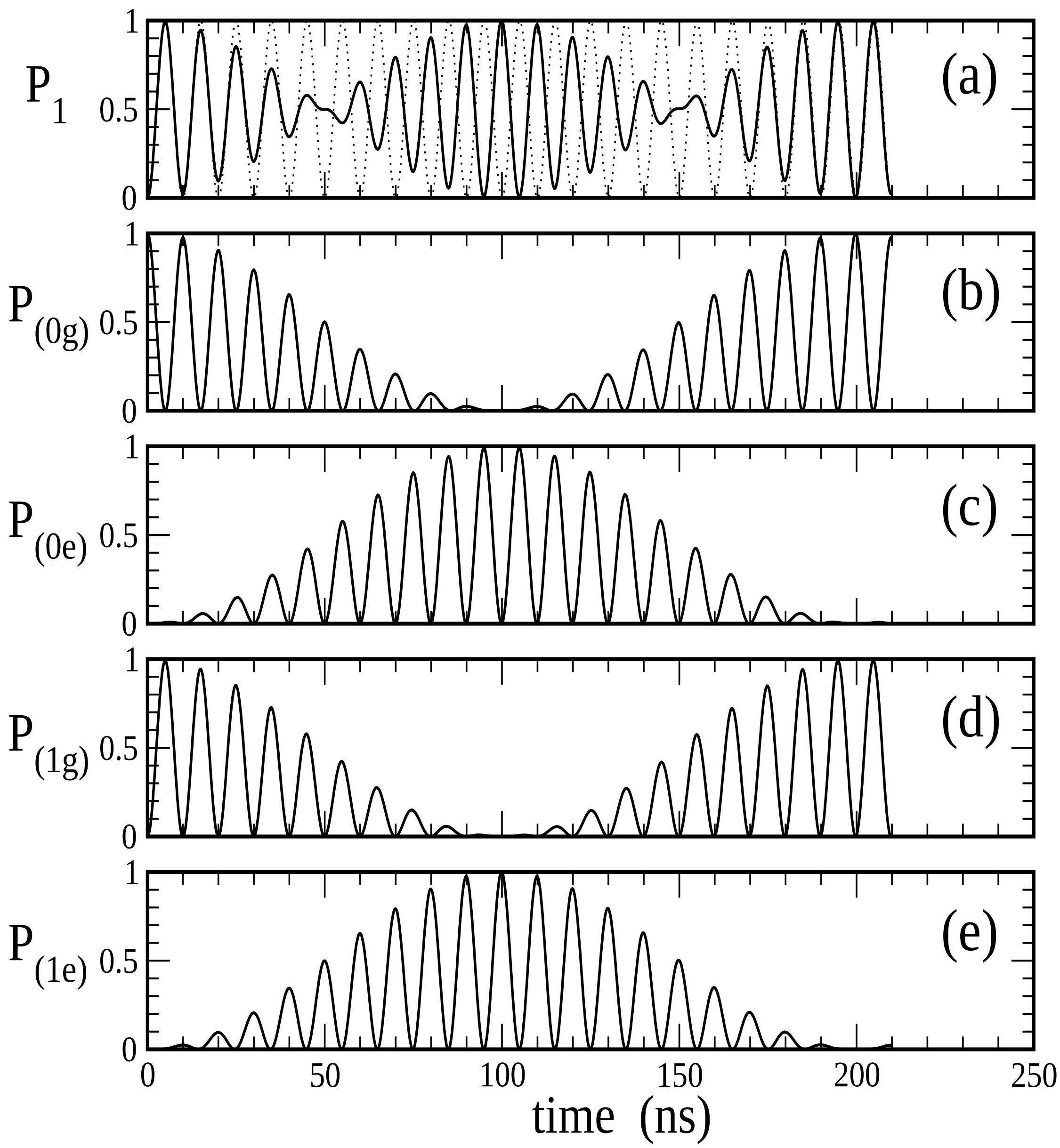

FIG. 1: Rabi oscillations of a resonantly coupled qubit-TLS system with $\varepsilon_{TLS} = \hbar\omega_{10}$. There is no mechanism for energy decay. Occupation probabilities of various states are plotted as functions of time. (a) P_1 is the occupation probability in the qubit state $|1\rangle$; (b) $P_{(0,g)}$ is the occupation probability in the state $|0,g\rangle$; (c) $P_{(0,e)}$ is the occupation probability of the state $|0,e\rangle$; (d) $P_{(1g)}$ is the occupation probability of the state $|1,g\rangle$; and (e) $P_{(1e)}$ is the occupation probability of the state $|1,e\rangle$. Notice the beating with frequency 2η. Throughout the paper, $\omega_{10}/2\pi = 10$ GHz. Parameters are chosen mainly according to the experiment in Ref. [15]: $\eta/\hbar\omega_{10} = 0.0005$, $g_{qb}/\hbar\omega_{10} = 0.01$, and $g_{TLS} = 0$. The dotted line in panel (a) shows the usual Rabi oscillations without resonant interaction, i.e. $\eta = 0$.

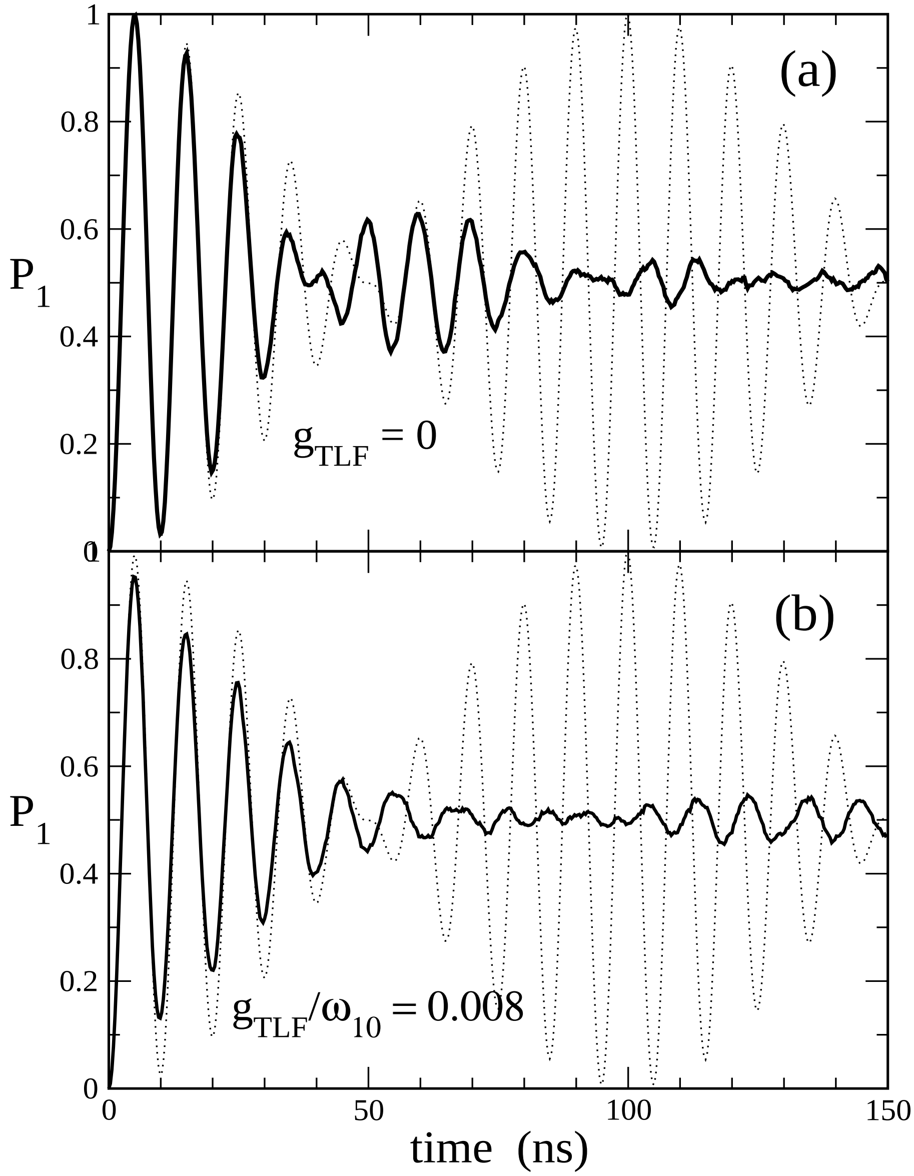

FIG. 2: Relaxation of Rabi oscillations due to energy decay of the TLS via phonon emission. Solid lines include the effect of the energy decay of the TLS with $\tau_{ph} = 10$ ns and the dotted lines have $\tau_{ph} = \infty$. These results are averaged over 100 runs in this figure and in all of the following figures. (a) No coupling between the TLS and the microwaves. (b) Direct coupling between the TLS and the microwaves with $g_{TLF}/\omega_{10} = 0.008$. The rest of the parameters are the same as in Fig.1. We note that the coupling between the TLS and microwaves degrades the qubit coherence.

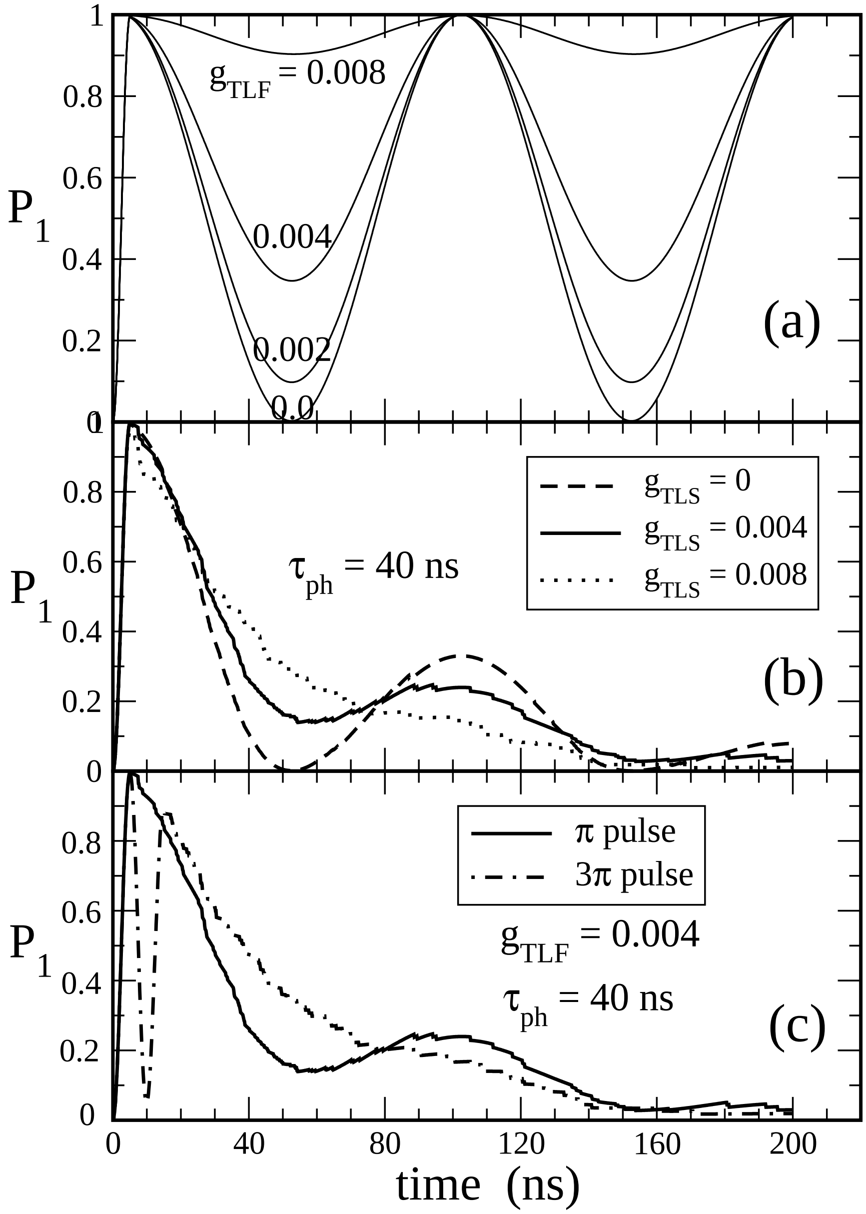

FIG. 3: The qubit-TLS system starts in its ground state at $t = 0$. A microwave π or 3π pulse (from 0 to 5 ns) puts the qubit-TLS system in the qubit excited state $|1\rangle$ that is a superposition of the two entangled states $|\psi_1'\rangle \equiv (|0,e\rangle + |1,g\rangle)/\sqrt{2}$ and $|\psi_2'\rangle \equiv (|0,e\rangle - |1,g\rangle)/\sqrt{2}$. After the microwaves are turned off, the occupation probability starts oscillating coherently. Values of g_{TLS} indicated in the figure are normalized by $\hbar\omega_{10}$. The rest of the parameters are the same as in Fig. 1. (a) No energy decay of the excited TLS, i.e., $\tau_{ph} = \infty$. Coherent oscillations with various values of g_{TLS}. (b) Oscillations following a π-pulse with $\tau_{ph} = 40$ ns and various values of g_{TLS}. (c) Oscillations following a π-pulse and a 3π-pulse with $\tau_{ph} = 40$ ns and $g_{TLS} = 0.004$. The dip in the dot-dash line is one and a half Rabi cycles.

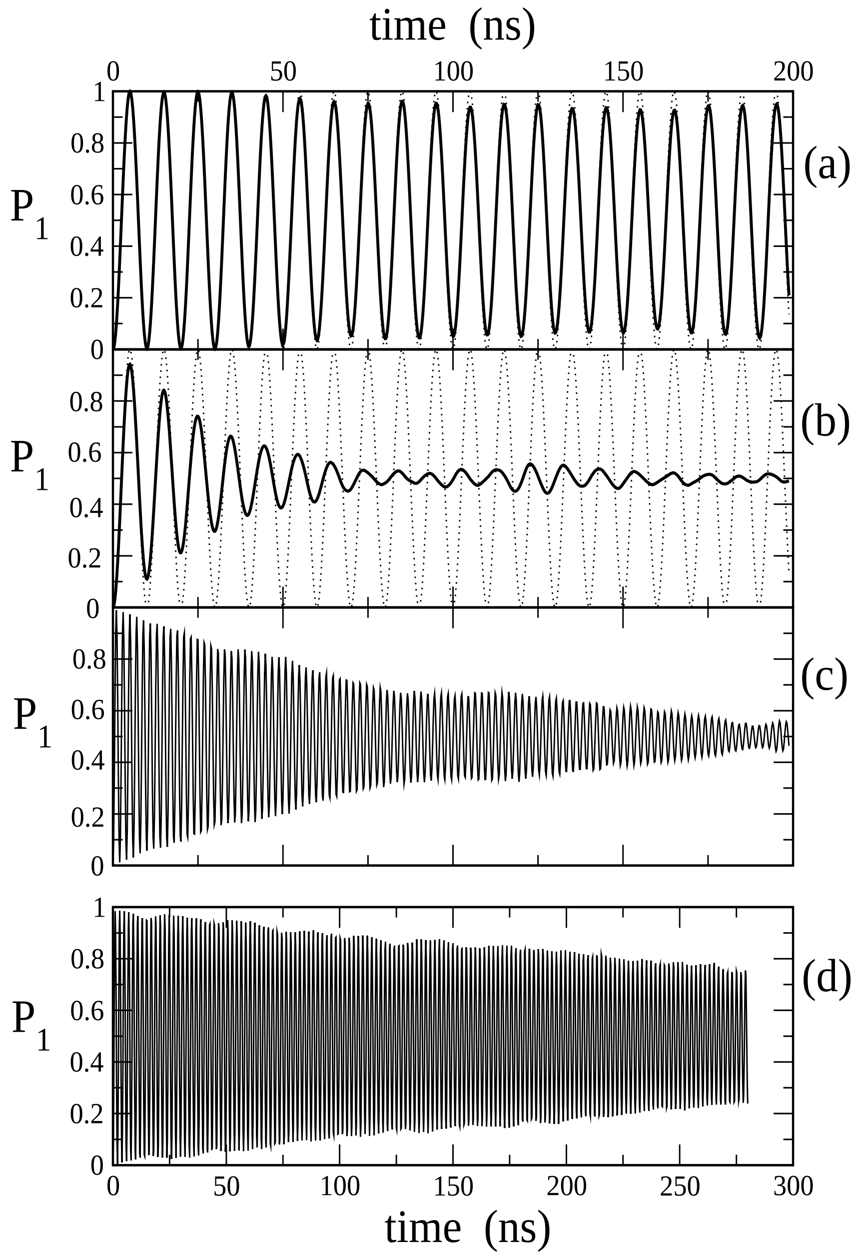

FIG. 4: Solid lines show the Rabi oscillation decay due to qubit level fluctuations caused by a single fluctuating two level system trapped inside the insulating tunnel barrier. The TLS produces random telegraph noise in I_o that modulates the qubit energy level splitting ω_{10}. (a) The level fluctuation $\delta\omega_{10}/\langle\omega_{10}\rangle = 0.001$. The characteristic fluctuation rate $t_{TLS}^{-1} = 0.6$ GHz. The Rabi frequency $f_R = 0.1$ GHz. The dotted lines show the usual Rabi oscillations without any noise source. (b) $\delta\omega_{10}/\langle\omega_{10}\rangle = 0.006$, $t_{TLS}^{-1} = 0.6$ GHz, and $f_R = 0.1$ GHz. The dotted lines show the usual Rabi oscillations without any noise source. (c) $\delta\omega_{10}/\langle\omega_{10}\rangle = 0.006$, $t_{TLS}^{-1} = 0.6$ GHz, and $f_R = 0.5$ GHz. (d) $\delta\omega_{10}/\langle\omega_{10}\rangle = 0.006$, $t_{TLS}^{-1} = 0.06$ GHz, and $f_R = 0.5$ GHz. Note that the scales of the horizontal axes in (a)-(c) are the same. They are different from that in (d).

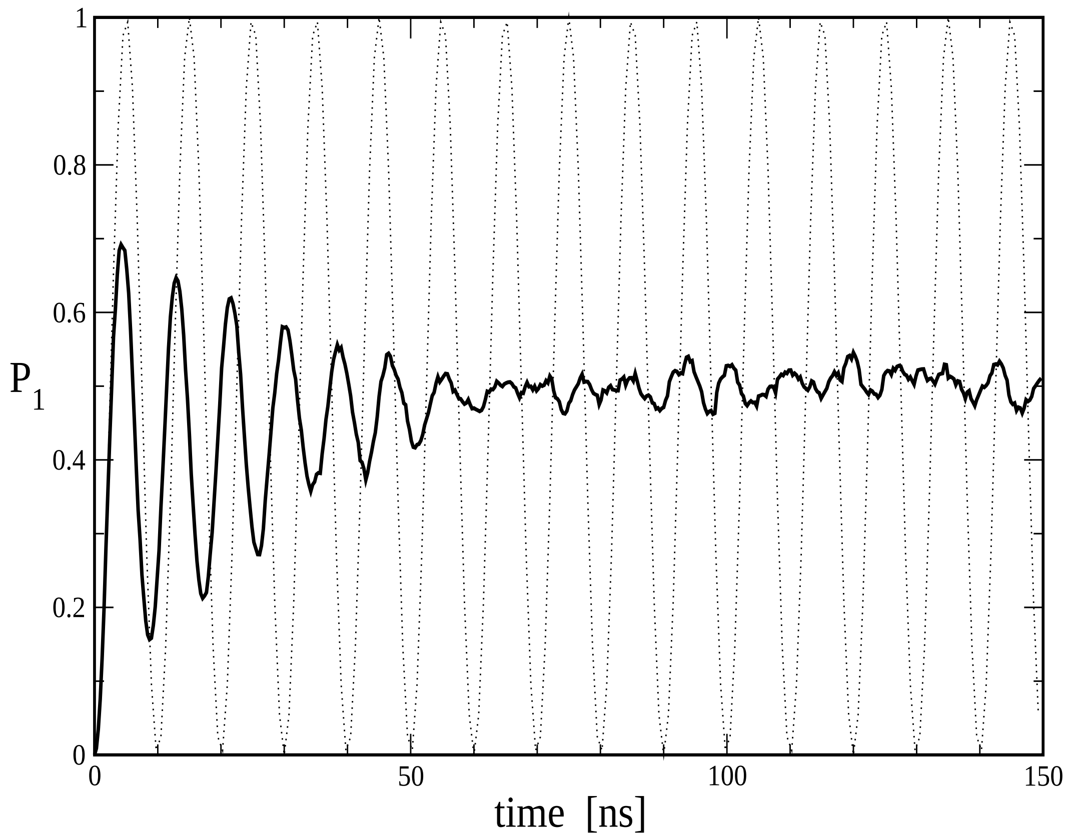

FIG. 5: Solid line represents Rabi oscillations in the presence of both TLS decoherence mechanisms: resonant interaction between the TLS and the qubit, and low frequency qubit energy level fluctuations caused by a single fluctuating TLS. The TLS couples to microwaves ($g_{TLS}/(\hbar\omega_{10}) = 0.008$) and the energy decay time for the TLS is $\tau_{ph} = 10$ ns, the same as in Fig. 2b. The size of the qubit level fluctuations is the same as in Fig. 4b. The dotted line shows the unperturbed Rabi oscillations.

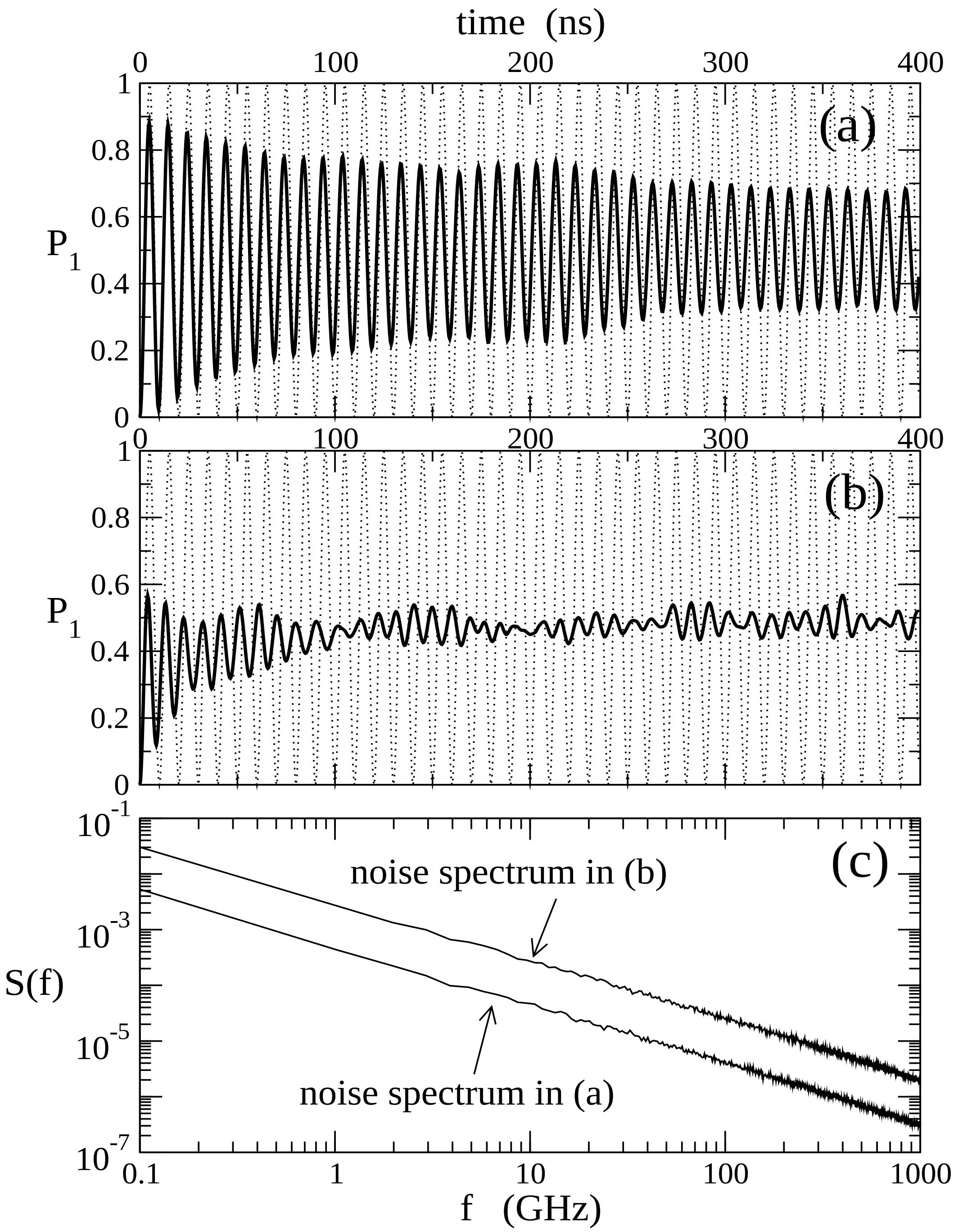

FIG. 6: (a)-(b): Rabi oscillations in the presence of 1/f noise in the qubit energy level splitting. Dotted curves show the Rabi oscillations without the influence of noise. Panel (c) shows the two noise power spectra $S(f) \equiv |\delta\omega_{10}(f)/\langle\omega_{10}\rangle|^2$ of the fluctuations in ω_{10} that were used to produce the solid curves in panels (a) and (b). Rabi frequency $f_R = 0.1$ GHz.